BUCKLING BEHAVIOUR OF THIN-WALLED COMPOSITE COLUMNS USING GENERALISED BEAM THEORY

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ABSTRACT

An extension of the existing Generalised Beam Theory (GBT), intended to make it applicable to orthotropic thin-walled structural members, is presented and discussed. The derived equations are, then, used to analyse the local and global buckling behaviour of FRP pultruded columns and the GBT results are validated through a comparison with numerical ones, obtained by finite strip analyses. Finally, a parametric study is carried out to investigate the variation of the bifurcation stress and buckling mode nature with (i) the column length and cross-section dimensions and (ii) the composite material properties.

KEYWORDS

Fiber reinforced plastics (FRP), Thin-walled columns, Generalised beam theory (GBT), Orthotropic material, Buckling behaviour, Bifurcation stresses, Local plate mode, Distortional mode.

INTRODUCTION

The structural use of composite materials can be traced back to the early 1960s and, since then, it has progressed through several stages. Although such materials have always been extensively employed in the aeronautical industry, civil engineering applications only became meaningful in recent years. Nevertheless, a growing number of companies have already emerged in this particular field and, moreover, they have managed to occupy a non negligible share in a market traditionally dominated by concrete and steel structures. This success is rooted in the fact that composite structural members combine (i) a nearly optimal weight/strength ratio with (ii) increasingly low fabrication costs and (iii) an efficient behaviour under aggressive environmental conditions. This last feature, in particular, makes composite materials ideally suited to be used in off-shore structures, chemical plants, etc.

This paper deals with a well defined type of composite structural members, namely thin-walled members with open cross-sections and manufactured, by pultrusion, from a polymer plastic matrix (e.g., epoxy)
reinforced with e-glass, s-glass, kevlar or carbon unidirectionally aligned fibers. Such members are usually designated as "FRP" pultruded members and their mechanical behaviour, which depends on the constituent properties and volume fraction, is characterised by (i) a linear elastic stress-strain relations (mostly with relatively low moduli), (ii) the absence of ductility (elastic behaviour up to collapse) and (iii) the presence of a distinct orthotropy. These mechanical properties clearly indicate that the FRP member behaviour is strongly affected by instability phenomena. Therefore, in order to assess the structural efficiency of such members, namely columns, beams and beam-columns, it is essential to investigate their (local and global) buckling behaviour. In particular, this involves the performance of stability analyses, aimed at identifying the relevant buckling modes and determining the corresponding bifurcation stresses.

The "Generalised Beam Theory" (GBT) was developed by Schardt (1989) and shown to be a rather powerful analytical tool to study the buckling behaviour of thin-walled (cold-formed) steel members (Davies, 1999). In fact, it provides a general and unified approach to obtain accurate solutions for a wide range of stability problems. By allowing the separation of the fundamental deformation modes, GBT offers possibilities not available through the use of numerical techniques, such as the finite element or finite strip methods (Davies et al, 1994). However, steel being an isotropic material, the existing GBT cannot be readily applied to orthotropic members (e.g., the FRP members under consideration), i.e., members made of materials displaying the stress-strain (constitutive) relations (it is assumed that $\nu_{xy}E_y=\nu_{yx}E_x$).

$$\begin{bmatrix}
\sigma_{xx} \\
\sigma_{ss} \\
\sigma_{xs}
\end{bmatrix} = 
\begin{bmatrix}
E_x/(1-\nu_{xx}v_{xx}) & v_{sx}E_x/(1-\nu_{sx}v_{xs}) & 0 \\
v_{sx}E_s/(1-\nu_{sx}v_{xs}) & E_s/(1-\nu_{sx}v_{xs}) & 0 \\
0 & 0 & G_{ss}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{ss} \\
\gamma_{xs}
\end{bmatrix},$$ (1)

The objectives of this work are (i) to extend the existing GBT, in order to make it applicable to orthotropic thin-walled structural members and (ii) to use the derived equations to analyse the local and global buckling behaviour of composite FRP columns with commonly used cross-sections. The GBT results are validated through a comparison with numerical ones, yielded by finite strip analyses. Finally, a parametric study is carried out to investigate the variation of the critical bifurcation stress and buckling mode nature with both (i) the column length and cross-section dimensions and (ii) the composite material properties.

**GBT FOR ORTHOTROPIC MATERIALS**

The simplifying assumptions adopted in the derivation of GBT for isotropic materials (Schardt, 1989), namely neglecting the membrane shearing strain and transversal extension, with the respect to the longitudinal extension values, remain perfectly valid in the context of the linear stability analysis of orthotropic columns. According with such assumptions, the relevant strain-displacement (kinematic) relations employed are defined by

$$\begin{align*}
\varepsilon_{xs}^F &= -zw_{ss} \\
\varepsilon_{ss}^F &= zw_{ss} \\
\gamma_{xs}^F &= -2zw_{ss} \\
\sigma_{ss}^M &= u_x + \frac{1}{2}(v^2_x + w^2_x),
\end{align*}$$ (2)

![Figure 1: Column geometry, coordinate system and displacement components.](image-url)
where \( x \) and \( s \) are coordinates along the length and mid line of the cross-section wall (see figure 1) and the superscripts \((\cdot)^{M}\) and \((\cdot)^{F}\) stand for the origin of the strain components (membrane or flexural).

In order to obtain a displacement representation compatible with the classical beam theory, Schardt (1989) prescribes that each displacement component \((u, v \text{ or } w)\) at any given point of the mid line of a cross-section comprised of \( n \) wall segments \( (\text{i.e., containing } n+1 \text{ nodes and/or degrees of freedom}) \) must be expressed as a combination of \( n+1 \) orthogonal functions \((u_{k})\), all of which vary linearly between consecutive nodes. One has, therefore,

\[
\begin{align*}
\text{u}(x,s) &= u_{k}\phi_{k,x} & \text{v}(x,s) &= v_{k}\phi_{k} & \text{w}(x,s) &= w_{k}\phi_{k}, \\
\end{align*}
\]  

where the summation convention applies to the subscript \( k \) \((1 \leq k \leq n+1)\) and \(\phi(x)\) is a "displacement amplitude function", defined along the column length. On the other hand, as a result of the assumptions underlying the derivation of GBT, all the remaining nodal displacement components \((v_{k} \text{ and } w_{k})\) are determined \textit{a priori}, on the basis of \(u_{k}\) values (Schardt, 1989). As a first step towards deriving the GBT fundamental equation for orthotropic materials, let us consider the variation of the column (membrane and flexural) strain energy, which is given by the expression

\[
\delta U = \int_{\Omega_{L}} \int_{-t/2}^{t/2} \left( \sigma_{xx}^{F} \delta \epsilon_{xx}^{F} + \sigma_{xs}^{F} \delta \gamma_{xs}^{F} + \sigma_{ss}^{F} \delta \epsilon_{ss}^{F} + \sigma_{xx}^{M} \delta \epsilon_{xx}^{M} \right) dzd\Omega, \tag{4}
\]

where \(\Omega_{L}\) is the combined area of the mid planes of the \( k \) plates forming the cross-section. Taking into account (i) the displacement approximations (3), (ii) the kinematic relations (2) and (iii) the constitutive relations (1), it is possible to rewrite the stress and strain variation components appearing in (4) as

\[
\begin{align*}
\sigma_{xx}^{F} &= -\frac{E_{x}}{1-\nu_{xs}} \left( zw_{k}\phi_{k,xx} + v_{xs}zw_{k,ss}\phi_{k} \right) & \delta \epsilon_{xx}^{F} &= -zw_{i}\delta \phi_{i,xx} \\
\sigma_{ss}^{F} &= -\frac{E_{s}}{1-\nu_{xs}} \left( zw_{k,ss}\phi_{k} + v_{xs}zw_{k,xx}\phi_{k} \right) & \delta \epsilon_{ss}^{F} &= -zw_{i,ss}\delta \phi_{i} \\
\sigma_{xs}^{F} &= -2zG_{xs}w_{k,\phi_{k}} & \delta \gamma_{xs}^{F} &= -2zw_{i,}\delta \phi_{i,} \\
\sigma_{xx}^{M} &= E_{x}u_{k}\phi_{k,xx} & \delta \epsilon_{xx}^{M} &= u_{i}\delta \phi_{i,xx} + (v_{i}v_{j}+w_{i}w_{j})\phi_{j,x}\delta \phi_{i,x},
\end{align*}
\]  

\(i.e.,\) to express \(\delta U\) as a function of the \((n+1)\) nodal displacements \(u_{k}\). Introducing (5)-(8) in (4) and performing the cross-section integration \( (w.r.t. \text{ the coordinates } s \text{ and } z) \), one is led to the expressions of the four terms comprising \(\delta U\), namely the terms due to the internal virtual work done by the (i) longitudinal bending normal stresses, (ii) transversal bending normal stresses, (iii) torsion shear stresses and (iv) membrane normal stresses. They are given, respectively, by \("b.c." – boundary condition terms)

\[
\begin{align*}
\delta U_{xx}^{F} &= \int_{\Omega_{L}} \int_{-t/2}^{t/2} \sigma_{xx}^{F} \delta \epsilon_{xx}^{F} dzd\Omega = \int_{\Omega_{L}} (E_{x}C_{xx}^{F}\phi_{k,xxxx} + v_{xs}E_{x}D_{sx}^{F}\phi_{k,xx}) \delta \phi_{i,xx} dx + \text{b.c.} \tag{9} \\
\delta U_{ss}^{F} &= \int_{\Omega_{L}} \int_{-t/2}^{t/2} \sigma_{ss}^{F} \delta \epsilon_{ss}^{F} dzd\Omega = \int_{\Omega_{L}} (E_{s}B_{ss}^{F}\phi_{k} + v_{xs}E_{s}D_{sx}^{F}\phi_{k,xx}) \delta \phi_{i,xx} dx + \text{b.c.} \tag{10} \\
\delta U_{xs}^{F} &= \int_{\Omega_{L}} \int_{-t/2}^{t/2} \sigma_{xs}^{F} \delta \gamma_{xs}^{F} dzd\Omega = -\int_{\Omega_{L}} (G_{xs}D_{sx}^{F}\phi_{k,xx}) \delta \phi_{i,xx} dx + \text{b.c.} \tag{11}
\end{align*}
\]
\[ \delta U_{xx}^M = \int_{\Omega}^{u_2} \int_{\Omega}^{L} \sigma_{xx} \delta \varepsilon_{xx} \, dz \, d\Omega = \int_{L}^{u_2} (E_x C_{ik} \phi_{k,xxxx} + X_{kij}(W_k \phi_{j,xx},x)) \delta \phi \, dx + \text{b.c.} \quad , \quad (12) \]

where the tensors appearing in the r.h.s., stemming from the cross-section integrations of the displacement components and their derivatives, are defined by the relations

\[ C_{ik}^\varphi = \frac{t^3}{12(1-\nu_x \nu_x)} \int_{S} w_i w_k \, ds \]
\[ D_{ik}^\varphi = \frac{t^3}{12(1-\nu_x \nu_x)} \int_{S} w_k w_{k,ss} \, ds \quad (13) \]
\[ B_{ik} = \frac{E_t t^3}{12(1-\nu_x \nu_x)} \int_{S} w_{k,ss} w_{k,ss} \, ds \]
\[ D_{ki}^\varphi = \frac{t^3}{12(1-\nu_x \nu_x)} \int_{S} w_k w_{k,ss} \, ds \quad (14) \]
\[ D_{ik} = \frac{t^3}{3} \int_{S} w_{k,k} \, ds \]  
\[ C_{ik} = t \int_{S} u_i u_k \, ds \]
\[ X_{kij} = t \int_{S} u_k (v_{ij} w_j + w_i) \, ds \quad . \quad (16) \]

Reassembling the terms of \( \delta U \) and noticing that the \( \delta \phi \) are arbitrary, it becomes possible to write the GBT fundamental (system of) equations for orthotropic materials, which reads

\[ E_x C_{ik} \phi_{k,xxxx} - G_{xs} D_{ik} \phi_{k,xx} + B_{ik} \phi_k + X_{kij}(W_k \phi_{j,xx})_x = 0 \quad . \quad (17) \]

This equation can be readily used to perform linear stability analyses of structural members subjected to arbitrary internal force combinations \( W_k = -E_x C_{ik} \phi_{k,xx} \). In the particular case of uniformly compressed members (columns), \( W_k \) corresponds to the axial force (compression) distribution resulting from the applied (axial) loads and includes the load/stress parameter, the critical value of which is sought.

When compared with the available GBT fundamental equation, derived in the context of isotropic materials (Davies et al., 1994), equations (17) have the advantage of enabling a more straightforward incorporation of the material properties (constants). Moreover, it is important to remark that:

(i) The tensors (13)-(16) differ from their isotropic counterparts (Davies et al., 1994) because of the presence of \( \nu_x \nu_x \) (instead of \( \nu^2 - C_{ik}^\varphi, D_{ik}^\varphi, B_{ik} \) and \( D_{ik}^\varphi \)) and \( E_x \) (instead of \( E - B_{ik} \)).

(ii) Equations (17) are slightly different from their isotropic counterparts, as (ii1) the first terms contain the longitudinal Young’s modulus \( E \) and (ii2) the second terms contain both the shear modulus \( G_{xs} \) and the constant \( \nu_x E_x \).

**LINEAR STABILITY ANALYSIS**

First of all, attention is called to the fact that, in general, the GBT system of differential equations (17) is coupled. In order to take full advantage of the GBT potential, matrices \([C]\), \([B]\) and \([D]\) (i.e., tensors \( C_{ik}, B_{ik} \) and \( D_{ik} \)) must be simultaneously diagonalised, by means of an orthogonal transformation defined by the solution of a standard eigenvalue problem. The corresponding eigenvector components are the nodal values of all the \( n+1 \) (modal) “warping functions” \( u_k \) (4 “rigid-body” and \( n-3 \) cross-section deformation functions), each of them related to three section properties \( (C_{kk}, B_{kk} \) and \( D_{kk} \) – diagonal components of \([C]\),
[B] and [D]). All the 2nd order effects taken into consideration by GBT are included in the last term of (17), which accounts for all the interactions between in-plane stresses and out-plane deformations. In fact, the non-null off diagonal components of matrix [X] (tensor $X_{ijk}$) are responsible for making system (17) coupled, which implies that the column buckling modes are linear combinations of the individual deformation modes. In geometrically linear (1st order) problems, the removal of this last term from system (17) makes it uncoupled.

In order to provide a better grasp of the concepts involved, let us consider the behaviour of a simply supported and uniformly compressed lipped channel column with cross-section dimensions: $b_{web}=20$ cm, $b_{flange}=10$ cm, $b_{lip}=4$ cm, $t=0.6$ cm. The column is made of an e-glass/epoxy composite material, manufactured in Brazil and characterised by the elastic constants: $E_x=17.236$ GPa, $E_y=6.894$ GPa, $G_{xy}=2.895$ GPa, $\nu_{xy}=0.36$, $\nu_{yx}=0.144$ (Nagahama, 2000). A 9 node discretization of the column cross-section was adopted (3 intermediate nodes) and the simultaneous matrix diagonalisation involved the tensors $C_{ik}$, $B_{ik}$ and $D_{ik}$. Figure 2 shows the shapes of the 7 (out of 9) most relevant cross-section deformation modes obtained and table 1 displays the values of (i) the cross-section modal properties ($C_{kk}$, $B_{kk}$ e $D_{kk}$) and (ii) geometric matrix components ($X_{ijk}$).

![Figure 2: Most relevant cross-section deformation mode shapes.](image)

<table>
<thead>
<tr>
<th>k</th>
<th>$C_{kk}$</th>
<th>$D_{kk}$</th>
<th>$B_{kk}$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28.8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1913.96</td>
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<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>9.258</td>
<td>0</td>
<td>-0.1149</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0.90303</td>
<td>0</td>
<td>-0.2223</td>
</tr>
<tr>
<td>4</td>
<td>51722.2</td>
<td>3.4558</td>
<td>0</td>
<td>0</td>
<td>9.258</td>
<td>0</td>
<td>-168.8</td>
<td>0</td>
<td>0.9910</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1.809</td>
<td>0.00286</td>
<td>0.00237</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.90303</td>
<td>0</td>
<td>-0.0466</td>
<td>0.03996</td>
</tr>
<tr>
<td>6</td>
<td>2.303</td>
<td>0.00408</td>
<td>0.00922</td>
<td>0</td>
<td>-0.1149</td>
<td>0</td>
<td>0.9910</td>
<td>0</td>
<td>-0.0502</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0.155</td>
<td>0.01555</td>
<td>0.18101</td>
<td>0</td>
<td>0</td>
<td>-0.2223</td>
<td>0</td>
<td>0.03996</td>
<td>0</td>
<td>-0.1709</td>
</tr>
</tbody>
</table>

For the first four modes (1 – longitudinal extension – $C_{11}$≡area; 2 and 3 – major/minor axis bending – $C_{22}$ and $C_{33}$≡major/minor moments of inertia; 4 – torsion – $C_{44}$≡warping constant and $D_{44}$≡St. Venant's constant), which are characterised by rigid-body motions, one has $B_{ik}=0$. On the other hand, the remaining three modes (5 – symmetric distortional, 6 – anti-symmetric distortional and 7 – local plate) involve deformations and, therefore, are associated to $B_{ik}≠0$. It should also be pointed out that no coupling is possible between the symmetric (even-number) and anti-symmetric (odd-number) modes, which stems from the fact that the corresponding $X_{ijk}$ off-diagonal components are null.
By incorporating the values presented in table 1 into the GBT equations (17) and assuming a sinusoidal buckling mode shape (with \( l \) half-wavelength), one is led to a system of algebraic equations defining an eigenvalue problem, the solutions of which are the roots of the corresponding characteristic equation (determinant). Figure 3(a) displays the variation of the bifurcation stress values \( \sigma_b \) with the column length \( L \) (logarithmic scale), considering (i) each mode individually (upper, lighter and numbered curves) and (ii) coupling between all the modes (lower and darker curve), and figure 3(b) makes it possible to estimate the "degree of participation" of each individual mode in the column (coupled) buckling mode.

![Figure 3: Variation, with \( L \), of (a) \( \sigma_b \) and (b) the modal participation in the column buckling mode.](image)

The observation of figures 3(a) and 3(b) leads to the following remarks:

(i) For \( L<140 \text{ cm} \), the "coupled curve" exhibits two local minima, corresponding, respectively, to bifurcation in a local plate mode (LPM) and in a symmetric distortional mode (DM). Depending on the value of the ratio \( b_c/t \), the local critical stress may correspond to either the LPM or the DM (LPM, in the particular case depicted). Notice also that the critical buckling mode is (i₁) a "pure" LPM (\( \#2 \)), for \( L<25 \text{ cm} \), (i₂) an "almost pure" DM (\( \#8 \) + a bit of \#9), for \( 60 \text{ cm}<L<140 \text{ cm} \), and (i₃) a mixed LPM-DM (\( \#5-\#7 \)), for \( 25 \text{ cm}<L<60 \text{ cm} \).

(ii) For \( 140 \text{ cm}<L<380 \text{ cm} \), buckling takes place in a mode designated as flexural-distortional (FDM), which combines major axis flexure (\( \#2 \)) with anti-symmetric distortion (\( \#8 \)) and a (negligible) torsion component (\#4). Although such mode may also be detected in isotropic columns, its relevance in orthotropic columns is much higher, particularly for rather low values of the ratio \( E_s/E_x \). In this particular case (\( E_s/E_x \approx 0.4 \)), the FDM is even critical for \( 220 \text{ cm}<L<380 \text{ cm} \).

(iii) For \( L>380 \text{ cm} \), \( \sigma_b \equiv \sigma_v \) decreases continuously and two (global) buckling modes may occur, namely (iii₁) a (major axis) flexural-torsional mode (FTM – \#2-\#4), for \( 380 \text{ cm}<L<840 \text{ cm} \), and (iii₂) a minor axis "pure" flexural mode (FM – \#3), for \( L>840 \text{ cm} \).

However, it is important to mention that the local minima (bifurcation stress values) do not depend on the half-wavelength number (\( m \)) displayed by the column buckling mode. The curves associated to \( m>1 \) can be obtained by a mere horizontal shift of the \( m=1 \) curve (the length ranges change accordingly).
Next, in order to validate the orthotropic GBT results obtained, they are compared with numerical ones, yielded by orthotropic finite strip analyses (Nagahama, 2000). Two columns are dealt with, namely (i) the lipped channel column analysed previously and (ii) an otherwise identical "Hat-section" (channel with outward lips) column. The GBT and finite strip (FS) results are presented and compared in figures 4(a) (lipped channel) and 4(b) ("Hat-section").

![Figure 4](image)

**Figure 4:** Comparison between GBT and finite strip results. (a) Lipped channel (b) "Hat-section"

Figures 4(a) and 4(b) show that the GBT and FS results are always practically identical (the difference never exceeds 3%). More specifically, the DM and FTM results are virtually identical and the (small) discrepancies are, basically, all restricted to column lengths associated to the vicinity of either (i) the LPM local minimum (GBT "below" FS) or (ii) the LPM-DM transition (GBT "above" FS). Moreover, it should be noticed that, unlike the lipped channel, the "Hat-section" does not exhibit a DM local minimum. This is due to the fact that the inward-outward lip change significantly decreases the cross-section torsional resistance (warping constant) and, therefore, the FTM bifurcation stress is lowered by a large amount.

**PARAMETRIC STUDY**

In this section, a limited parametric study is performed, aimed at investigating the variation of the bifurcation stress values and buckling mode nature with the (i) FRP properties and (ii) the cross-section dimensions. Concerning the first aspect, the previous lipped channel column is considered and assumed made of four different fiber reinforced plastics, all with an epoxy matrix and unidirectionally aligned fibers occupying 60% of the volume.

**TABLE 2**

<table>
<thead>
<tr>
<th>Fibers</th>
<th>$E_x$ (GPa)</th>
<th>$E_y$ (GPa)</th>
<th>$G_{xy}$ (GPa)</th>
<th>$\nu_{xy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>e-glass ($G_1$)</td>
<td>40</td>
<td>8</td>
<td>4</td>
<td>0.25</td>
</tr>
<tr>
<td>e-glass ($G_2$)</td>
<td>17.2</td>
<td>6.9</td>
<td>2.9</td>
<td>0.36</td>
</tr>
<tr>
<td>kevlar (K)</td>
<td>75</td>
<td>6</td>
<td>2</td>
<td>0.34</td>
</tr>
<tr>
<td>hs carbon ($C_1$)</td>
<td>140</td>
<td>10</td>
<td>5</td>
<td>0.30</td>
</tr>
<tr>
<td>hm carbon ($C_2$)</td>
<td>180</td>
<td>8</td>
<td>5</td>
<td>0.30</td>
</tr>
</tbody>
</table>
These fibers are made of either (i) e-glass (G1), (ii) kevlar (K), (iii) high strength carbon (C1) or (iv) high modulus carbon (C2), and their elastic constant values are presented in table 2 (Datoo, 1991). For comparison purposes, the case of the e-glass/epoxy composite (G2) used earlier, with lower elastic constant values (the fiber volume fraction is smaller) is also addressed.

Figure 5: Variation of $\sigma_b$ with $L$, for five different FRP materials.

Figure 5 depicts the variation of $\sigma_b$ with $L$, for the five different composite columns. One observes that:

(i) The $\sigma_{bL}^P$ values are 37 MPa (G2), 59 MPa (G1 and K) and 108 MPa (C1 and C2). Notice that, in spite of the distinct material properties, the $\sigma_{bL}^P$ values are identical for (i1) G1 and K, and (i2) C1 and C2.

(ii) The $\sigma_{bD}^P$ values are 43 MPa (G2), 69 MPa (G1), 77 MPa (K), 138 MPa (C1) and 140 MPa (C2). Notice that the $\sigma_{bD}^P$ values are practically for C1 and C2.

(iii) The (global) $\sigma_{bT}^P$ values essentially increase with $E_x$ (G2 $\rightarrow$ G1 $\rightarrow$ K $\rightarrow$ C1 $\rightarrow$ C2).

(iv) Apart from a small horizontal shift, the C1 and C2 curves practically coincide, which means that their local and global buckling behaviours are virtually identical.

(v) Decreasing the fiber volume fraction (G1 $\rightarrow$ G2) induces a similar reduction in the $\sigma_{bL}^P$ and $\sigma_{bD}^P$ values ($\sigma_{bL}^{P,G2}/\sigma_{bL}^{P,G1}=37/59=\sigma_{bD}^{P,G2}/\sigma_{bD}^{P,G1}=43/69=0.625$).

Next, it is intended to investigate the influence of the lipped channel column cross-section dimensions on the nature of the local critical buckling mode and corresponding $\sigma_b$ value. Taking into consideration the cross-section proportions of the commonly used pultruded columns, the following dimension ratios are considered: (i) $b_w/t=20$ and 40, (ii) $b/b_w=0.25$, 0.50 and 0.75 and (iii) $b/b_w=0.05$ to 0.40. Figure 6 shows the variation of the $\sigma_b$ with $b_l/b_w$, for the LPM (dotted line), DM (solid-dashed lines).

Figure 6: Variation of $\sigma_b$ with $b_l/b_w$ and $b_l/b_w$. (a) $b_w/t=20$ (b) $b_w/t=40$
From this limited parametric study, it is possible to conclude that:

(i) As expected, the $\sigma_{\text{LP}}^b$ values are practically independent of the lips width (the dotted lines are practically horizontal). They only depend on the web (mainly) and flanges widths.

(ii) The $\sigma_{\text{D}}^b$ vs. $b_1/b_w$ curves display maximum values for (ii1) $b_1/b_w=0.16$ ($b_f/b_w=0.25$), (ii2) $b_1/b_w=0.24$ ($b_f/b_w=0.50$) and (ii3) $b_1/b_w=0.38$ ($b_f/b_w=0.75$). Notice also that, for (unrealistically) large $b_1/b_w$ and when $b_1/b_w < 0.75$, $\sigma_{\text{D}}^b$ is not associated with a local minimum (dashed portions of the curves).

(iii) For the lower plate slenderness value ($b_w/t=20$), the local critical stress is always associated to the DM ($\sigma_{\text{cr}}=\sigma_{\text{DM}}^b$). For the upper plate slenderness value ($b_w/t=40$), on the other hand, it may be associated to either the DM ($\sigma_{\text{cr}}=\sigma_{\text{DM}}^b$) or the LPM ($\sigma_{\text{cr}}=\sigma_{\text{LP}}^b$), depending on the $b_1/b_w$ value.

(iv) The lip width values associated to critical DM-LPM transitions (white circles in figure 6(b) have an obvious practical interest, as they lead to the definition of "optimally efficient" lips (stiffeners). Therefore, design formulae to estimate such width values would be rather useful.

**CONCLUDING REMARKS**

(i) The existing GBT, developed in the context of isotropic materials, was extended in order to enable its application to orthotropic thin-walled structural members.

(ii) The derived GBT equations were first validated, by means of a comparison with numerical results obtained through finite strip analyses, and, then, applied to study the local and global buckling behaviour of lipped channel FRP columns. In particular, a mixed flexural-distortional buckling mode was identified, which does not appear in isotropic (e.g., cold-formed steel) columns. Such mode was shown to be critical for intermediate length columns.

(iii) A limited parametric study was carried out to investigate the influence of (iii1) the composite material properties and (iii2) the column length and cross-section dimensions on the critical bifurcation stress and buckling mode nature.

(iv) The buckling behaviour of five geometrically identical lipped channel columns made of different fiber reinforced plastics, all with an epoxy matrix and unidirectionally aligned glass, kevlar or carbon fibers, was investigated. The analysis unveiled markedly different behaviours and the results obtained showed the carbon fibers to be the ones providing the highest local and global buckling resistance.

(v) Design formulae to evaluate the widths of "optimally efficient" lips (i.e., widths associated to critical DM-LPM transitions) would be very useful in practice and, therefore, should be sought.

**References**


